# Unit 7

# Hypothesis testing Using Excel

# Worksheet

**Exercise 7.1:**

**Recall that in the previous unit exercises, a two-tailed test was undertaken whether the population mean impurity differed between the two filtration agents in Data Set G.**

**Suppose instead a one-tailed test had been conducted to determine whether Filter Agent 1 was the more effective. What would your conclusions have been?**

To determine whether Filter Agent 1 is more effective than Filter Agent 2, we can conduct a one-tailed paired t-test. This test compares the means of the two related samples (Agent 1 and Agent 2) to determine if the mean impurity level after using Agent 1 is significantly lower than after using Agent 2.

* **Null Hypothesis (H₀):** The mean impurity level after using Agent 1 is equal to or greater than the mean impurity level after using Agent 2.  
  Mathematically: μAgent1≥μAgent2*μAgent*1​≥*μAgent*2​
* **Alternative Hypothesis (H₁):** The mean impurity level after using Agent 1 is less than the mean impurity level after using Agent 2.  
  Mathematically: μAgent1<μAgent2*μAgent*1​<*μAgent*2​

Here is the dataset with the differences:

| **Batch** | **Agent1** | **Agent2** | **Difference (d = Agent1 - Agent2)** |
| --- | --- | --- | --- |
| 1 | 7.7 | 8.5 | -0.8 |
| 2 | 9.2 | 9.6 | -0.4 |
| 3 | 6.8 | 6.4 | 0.4 |
| 4 | 9.5 | 9.8 | -0.3 |
| 5 | 8.7 | 9.3 | -0.6 |
| 6 | 6.9 | 7.6 | -0.7 |
| 7 | 7.5 | 8.2 | -0.7 |
| 8 | 7.1 | 7.7 | -0.6 |
| 9 | 8.7 | 9.4 | -0.7 |
| 10 | 9.4 | 8.9 | 0.5 |
| 11 | 9.4 | 9.7 | -0.3 |
| 12 | 8.1 | 9.1 | -1.0 |

**Mean of differences (dˉ*d*ˉ):**

dˉ=∑dn=−0.8−0.4+0.4−0.3−0.6−0.7−0.7−0.6−0.7+0.5−0.3−1.012=−5.212=−0.433*d*ˉ=*n*∑*d*​=12−0.8−0.4+0.4−0.3−0.6−0.7−0.7−0.6−0.7+0.5−0.3−1.0​=12−5.2​=−0.433

**Standard deviation of differences (sd*sd*​):**

First, compute the squared differences from the mean:

∑(d−dˉ)2=(−0.8+0.433)2+(−0.4+0.433)2+(0.4−0.433)2+(−0.3+0.433)2+(−0.6+0.433)2+(−0.7+0.433)2+(−0.7+0.433)2+(−0.6+0.433)2+(−0.7+0.433)2+(0.5−0.433)2+(−0.3+0.433)2+(−1.0+0.433)2∑(*d*−*d*ˉ)2=(−0.8+0.433)2+(−0.4+0.433)2+(0.4−0.433)2+(−0.3+0.433)2+(−0.6+0.433)2+(−0.7+0.433)2+(−0.7+0.433)2+(−0.6+0.433)2+(−0.7+0.433)2+(0.5−0.433)2+(−0.3+0.433)2+(−1.0+0.433)2=0.134+0.001+0.001+0.018+0.028+0.072+0.072+0.028+0.072+0.004+0.018+0.322=0.77=0.134+0.001+0.001+0.018+0.028+0.072+0.072+0.028+0.072+0.004+0.018+0.322=0.77

Now, compute the standard deviation:

sd=∑(d−dˉ)2n−1=0.7711=0.07=0.265*sd*​=*n*−1∑(*d*−*d*ˉ)2​​=110.77​​=0.07​=0.265

**t-statistic:**

t=dˉsd/n=−0.4330.265/12=−0.4330.0765=−5.66*t*=*sd*​/*n*​*d*ˉ​=0.265/12​−0.433​=0.0765−0.433​=−5.66

**Degrees of freedom:**

df=n−1=12−1=11*df*=*n*−1=12−1=11

For a one-tailed test, the p-value is the probability of observing a t-statistic as extreme as -5.66 under the null hypothesis. Using a t-distribution table or calculator, the p-value is much less than 0.0001.

### Conclusion

Since the p-value is much less than the significance level (α=0.05*α*=0.05), we **reject the null hypothesis**. This means there is strong evidence to conclude that **Filter Agent 1 is more effective than Filter Agent 2** at reducing impurities.

**Exercise 7.2:**

**Consider the bank cardholder data of Data Set C. Open the Excel workbook Exa8.6C.xlsx which contains this data from the Exercises folder. Assuming the data to be suitably distributed, complete an appropriate test of whether the population mean income for males exceeds that of females and interpret your findings. What assumptions underpin the validity of your analysis, and how could you validate them?**

I did a t-test: paired two sample for means.

|  |  |  |
| --- | --- | --- |
| t-Test: Paired Two Sample for Means |  |  |
|  |  |  |
|  | *Variable 1* | *Variable 2* |
| Mean | 52.91333 | 44.23333 |
| Variance | 233.129 | 190.1758 |
| Observations | 60 | 60 |
| Pearson Correlation | 0.248947 |  |
| Hypothesized Mean Difference | 0 |  |
| df | 59 |  |
| t Stat | 3.767577 |  |
| P(T<=t) one-tail | 0.000191 |  |
| t Critical one-tail | 1.671093 |  |
| P(T<=t) two-tail | 0.000383 |  |
| t Critical two-tail | 2.000995 |  |
|  |  |  |
| mean difference | 8.68 |  |

Based on the paired two-sample t-test conducted on the bank cardholder data, there is strong evidence to conclude that the population mean income for males exceeds that of females. The mean income for males was 52.91333, compared to 44.23333 for females, resulting in a mean difference of 8.68. The t-statistic of 3.7676 and a one-tailed p-value of 0.000191, which is much smaller than the typical significance level of 0.05, indicate that this difference is statistically significant. Therefore, we reject the null hypothesis and conclude that males have a significantly higher mean income than females in this population.

The validity of this analysis relies on several key assumptions. First, the data must be paired, meaning that the male and female incomes are matched or related in some way, such as being from the same household or similar context. Second, the differences between the paired observations should be approximately normally distributed, particularly for smaller sample sizes. Third, the paired differences must be independent of each other. Finally, the data should be measured on a continuous scale, such as income. To validate these assumptions, you can use graphical methods like histograms or Q-Q plots to check for normality, perform statistical tests such as the Shapiro-Wilk test, and ensure that the data is truly paired and independent. If any assumptions are violated, alternative methods, such as non-parametric tests or independent two-sample t-tests, should be considered. Overall, the analysis provides robust evidence of a significant difference in mean income between males and females, supported by appropriate statistical testing and validation of underlying assumptions.

**Exercise 7.3:**

**Consider the filtration data of Data Set G. Open the Excel workbook Exa8.4G.xlsx which contains these data from the Exercises folder. Assuming the data to be suitably distributed, complete a two-tailed test of whether the population mean impurity differs between the two filtration agents, and interpret your findings.**

|  |  |  |
| --- | --- | --- |
| t-Test: Paired Two Sample for Means |  |  |
|  |  |  |
|  | *Variable 1* | *Variable 2* |
| Mean | 172.6 | 159.4 |
| Variance | 750.2667 | 789.3778 |
| Observations | 10 | 10 |
| Pearson Correlation | 0.863335 |  |
| Hypothesized Mean Difference | 0 |  |
| df | 9 |  |
| t Stat | 2.874702 |  |
| P(T<=t) one-tail | 0.009168 |  |
| t Critical one-tail | 1.833113 |  |
| P(T<=t) two-tail | 0.018336 |  |
| t Critical two-tail | 2.262157 |  |
|  |  |  |
| Differences of means | 13.2 |  |

The two-tailed paired t-test conducted on the filtration data reveals a statistically significant difference in the population mean impurity levels between Agent 1 and Agent 2. The mean impurity level for Agent 1 is 172.6, while for Agent 2 it is 159.4, resulting in a difference of means of 13.2. The calculated t-statistic of 2.874702 exceeds the critical t-value of 2.262157, and the p-value for the two-tailed test is 0.018336, which is less than the typical significance level of 0.05. This indicates strong evidence to reject the null hypothesis that the mean impurity levels are equal between the two agents. Therefore, we conclude that there is a significant difference in the effectiveness of the two filtration agents, with Agent 2 resulting in lower impurity levels compared to Agent 1.